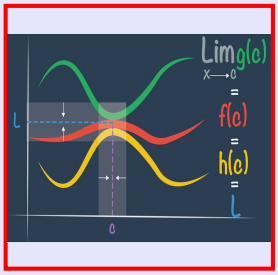
Math 261

Spring 2023

Lecture 4



Feb 19-8:47 AM

Given $f(x) = x^2 - 4x$

1) Y-Int $\rightarrow x=0$, $f(0) = 0^2 - 4(0) = 0 \Rightarrow (0,0)$

2) X-Ints $\rightarrow y=0$, $f(x)=0$, $x^2-4x=0 \Rightarrow (0,0) \,\&\, (4,0)$

$x(x-4)=0 \quad x=0,\ x=4$

3) Simplify, and evaluate $g(x) = \dfrac{f(x+h)-f(x)}{h}$ for $h=0$.

$$\frac{f(x+h)-f(x)}{h} = \frac{(x+h)^2 - 4(x+h) - (x^2-4x)}{h}$$

$$= \frac{\cancel{x^2} + 2xh + h^2 - \cancel{4x} - 4h - \cancel{x^2} + \cancel{4x}}{h}$$

$$= \frac{2xh + h^2 - 4h}{h} = \frac{\cancel{h}(2x+h-4)}{\cancel{h}}$$

$$= 2x + h - 4$$

For $h=0 \rightarrow \boxed{g(x) = 2x-4}$

Feb 9-8:48 AM

Graph $f(x)$ & $g(x)$

$f(x) = x^2 - 4x$

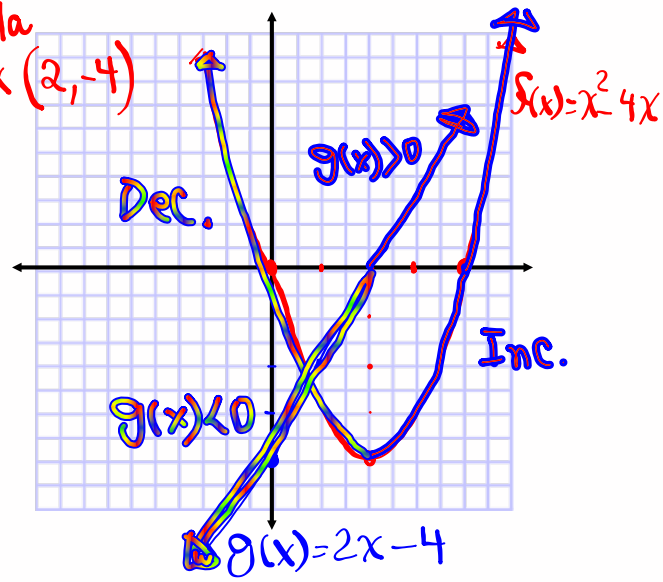
Parabola

Vertex $(2,-4)$

$g(x) = 2x - 4$

Y-Int $(0,-4)$

$m = 2 = \frac{2}{1}$



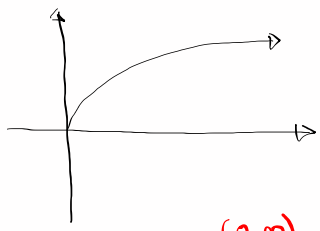
Feb 9-8:56 AM

Given $f(x) = \sqrt{x}$

1) Y-Int $\rightarrow (0,0)$

2) x-Int $\rightarrow (0,0)$

3) Graph $f(x)$



4) Discuss increasing & Decreasing. Increasing $(0,\infty)$

5) Simplify and Find $g(x) = \frac{f(x+h) - f(x)}{h}$ for $h=0$

$$\frac{f(x+h)-f(x)}{h} = \frac{\sqrt{x+h} - \sqrt{x}}{h} = \frac{(\sqrt{x+h} - \sqrt{x})(\sqrt{x+h} + \sqrt{x})}{h(\sqrt{x+h} + \sqrt{x})}$$

$(A - B)(A + B)$

$A^2 - B^2$

$$= \frac{(\sqrt{x+h})^2 - (\sqrt{x})^2}{h(\sqrt{x+h} + \sqrt{x})} = \frac{\cancel{x} + \cancel{h}^{1} - \cancel{x}}{\cancel{h}(\sqrt{x+h} + \sqrt{x})} = \frac{1}{\sqrt{x+h} + \sqrt{x}}$$

For $h=0 \rightarrow g(x) = \frac{1}{2\sqrt{x}}$ $\quad g(x) > 0$

Since $g(x) > 0$, $f(x)$ must be increasing.

Feb 9-9:02 AM

Given $f(x) = \frac{1}{x}$

1) Y-Int $\rightarrow x = 0$ $\quad \frac{1}{0}$ undefined $\rightarrow$ None

2) x-Int. $\rightarrow y = 0 \rightarrow f(x) = 0 \rightarrow \frac{1}{x} = 0 \rightarrow$ No Solution

$\rightarrow$ None

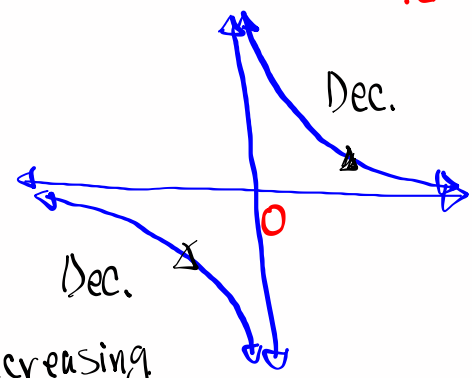
3) Graph $f(x)$

as $x \rightarrow 0^+$, $y \rightarrow \infty$

as $x \rightarrow 0^-$, $y \rightarrow -\infty$

as $x \rightarrow \infty$, $y \rightarrow 0$

as $x \rightarrow -\infty$, $y \rightarrow 0$



4) Discuss increasing & Decreasing

$(-\infty, 0) \cup (0, \infty)$

Feb 9-9:15 AM

5) Simplify and find $g(x) = \frac{f(x+h) - f(x)}{h}$ for $h = 0$.

$$\frac{f(x+h) - f(x)}{h} = \frac{\frac{1}{x+h} - \frac{1}{x}}{h} = \frac{x\cancel{(x+h)} \cdot \frac{1}{\cancel{x+h}} - \cancel{x}(x+h) \cdot \frac{1}{\cancel{x}}}{h \cdot x(x+h)}$$

LCD $= x(x+h)$

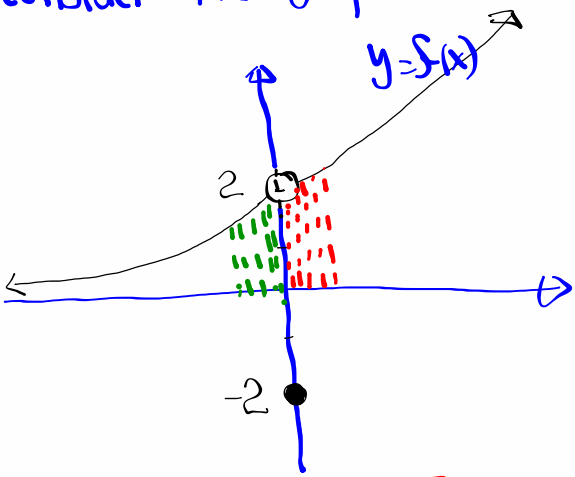
$$= \frac{x - (x+h)}{h\,x(x+h)} = \frac{\cancel{x} - \cancel{x} - \cancel{h}}{\cancel{h}\,x(x+h)} = \frac{-1}{x(x+h)}$$

For $h = 0 \rightarrow g(x) = \frac{-1}{x^2}$ $\quad \cancel{g(x) > 0}$ $\quad \boxed{g(x) < 0}$

when $g(x) < 0$, $f(x)$ must be decreasing.

Feb 9-9:22 AM

Consider the graph below:



1) $f(0) = \boxed{-2}$

2) as $x \to -\infty$, $y \to \boxed{0}$

3) as $x \to \infty$, $y \to \boxed{\infty}$

4) as $x \to 0^+$, $y \to \boxed{2}$

5) as $x \to 0^-$, $y \to \boxed{2}$

Feb 9-9:30 AM

Class QZ 1

Solve $3x^2 - 5x = 8$ using the quadratic formula.

$3x^2 - 5x - 8 = 0$

$a = 3$
$b = -5$
$c = -8$

$b^2 - 4ac = (-5)^2 - 4(3)(-8) = 25 + 86 = 121$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-5) \pm \sqrt{121}}{2(3)} = \frac{5 \pm 11}{6}$$

$$x = \frac{5 + 11}{6} = \frac{16}{6} = \frac{8}{3}$$

$$x = \frac{5 - 11}{6} = \frac{-6}{6} = -1$$

$\left\{-1, \frac{8}{3}\right\}$

Feb 9-9:39 AM